

# EXAM 3 B

PRINT NAME

57 (1) Find the most general Antiderivative for each of the following

a)  $f(x) = x^3 - 4x^2 + 17$

b)  $g(T) = \sin T - 2\cos T$

c)  $h(x) = \sqrt[5]{x^3} + \sqrt[3]{x^5}$

(2) Given that the graph of  $f$  passes through the point  $(1, 5)$  and the slope of its tangent line @  $(x, f(x))$  is  $2x - 1$ , find  $f(2)$

Q] (3) If an object moves so that the acceleration at time  $t$  is given by  $a(t) = t^2 + 3 \cos t$  in  $m/s^2$  and its initial velocity is  $3 m/s$  and its initial position is  $2m$

a) Find the velocity as a function of time  $t$  (be as specific as possible)

$v(t) =$

b) Find the position as a function of time  $t$  (be specific)

$s(t) =$

7] (4) Find the Absolute Maximum and Minimum Values of the function  $f(x) = x^2 + \frac{2}{x}$  on the interval  $[\frac{1}{2}, 2]$ . Show work.

[15] (5) Find the following Limits. Justify your answers by clearly showing steps.

a)  $\lim_{x \rightarrow -\infty} \frac{3x^2 - x - 1}{4x^2 + 7}$

b)  $\lim_{x \rightarrow \infty} \frac{\sqrt{1 + 4x^2}}{4 + x}$

c)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x + 1} - x)$

[12] (6) Let  $f(x) = x^3 - 3x + 1$

a) Find all critical numbers for  $f$

b) Determine intervals of increase and decrease

c) Determine intervals of Concavity

137 ⑦ Given a "mystery" function  $y = f(x)$   
 Sketch its graph based on the following information. Be sure to  
 Label all intercepts, asymptotes, Local Extrema, and/or Inflection points.

A) Domain =  $\mathbb{R} = (-\infty, \infty)$

B) y int  $f(0) = 0$   
 x int  $f(x) = 0 \Rightarrow x = 0$

C)  $f(-x) = -f(x)$

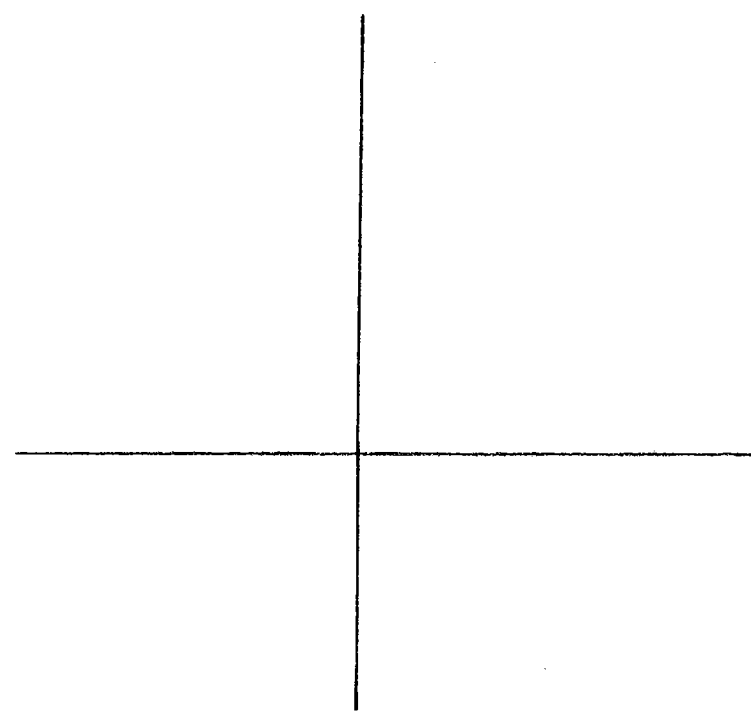
D)  $\lim_{x \rightarrow \infty} f(x) = 1$   
 $\lim_{x \rightarrow -\infty} f(x) = -1$

E) + F)

	$f'$
$(-\infty, \infty)$	+

G)

	$f''$
$(-\infty, 0)$	+
$(0, \infty)$	-



[10] ⑧ Find the point on the line  $y = 2x - 3$  that is closest to the origin. Show work for credit.

9) a) State Rolle's Theorem

b) State the Mean Value Theorem

c) Use the Intermediate Value Theorem to show that  $f(x) = x^5 + 10x - 3$  has a real root on  $[0, 1]$ .

⑨ d) Use either Rolle's Theorem (or if you prefer the Mean Value Theorem) to show that  $f$  has only one real root.

Hint: Suppose  $f$  has two roots  $a$  and  $b$  with  $a < b$ . Apply Rolle's Theorem to  $f$  and reach a contradiction. Be sure to explain why all the hypotheses of the theorem ~~are~~ are valid.

Bonus Use Rolle's Thm to Prove the Mean Value Theorem