

EXAM 2c

Print Name

① Differentiate. Do Not Simplify.

a) $y = \sqrt{x} + 3\pi$

b) $f(x) = 3x^5 + 4x + 10$

c) $g(x) = \frac{\sqrt{10}}{x^8}$

d) $h(\theta) = \theta \csc \theta$

e)
$$y = \frac{x^3}{x^2 + \tan x}$$

f)
$$F(T) = (T^6 + 5T^4)^{100}$$

g)
$$G(x) = (2x^3 - 5)^{-4/3} (x^5 + 4x)^{5/2}$$

② Use Implicit Differentiation to find an equation of the tangent line to the curve

$$2(x^2 + y^2) = 5(x^2 - 5y^2)$$

@ the point (3, 1)

③ Given $f(x) = \cos x$

Find the Linearization for $f(x)$ @ $a = \frac{2\pi}{3}$
i.e. the Linear approximation $L(x)$

④ The equation of motion of an object is

given by $s(t) = 2 \sin 5t$ where s is in meters and t is in seconds

a) Find the velocity as a function of time t

b) Find the acceleration as a function of time t

⑤ Assume that a function $y = f(x)$ is differentiable for all x

Recall: f is an EVEN function if $f(-x) = f(x)$ for all x

and f is an ODD function if $f(-x) = -f(x)$ for all x

Use the Chain Rule to show that

if f is ODD, then f' is EVEN

⑥ For specific constants A and B ,
the function $y = A \sin x + B \cos x$

solves the differential equation

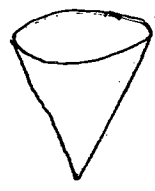
$$y'' + y' - 2y = \sin x$$

Find the numbers $A + B$.

Show work to justify
your answers.

7) A water tank in the shape of an inverted circular cone with base radius 3 meters and height 9 meters has water pumped into it at a rate of $2 \text{ m}^3/\text{min}$. Find the rate @ which the water level is rising when the water is 1 meter deep.

Note $V = \frac{1}{3} \pi r^2 h$



⑧ Use your knowledge of the Quotient Rule (or Chain, Product, & Power Rules) to prove

$$\frac{d}{dx} (\cot x) = -\csc^2 x$$

⑨ Find a formula for $f^{(n)}(x)$
where $f(x) = (1+2x)^{-1}$

Bonus

a) Finish the following statement of the product rule

$$(f(x)g(x))' =$$

b) Use the definition of the derivative, i.e. $P'(x) = \lim_{h \rightarrow 0} \frac{P(x+h) - P(x)}{h}$

to Prove the Product Rule from (a)

You must clearly show ALL steps.